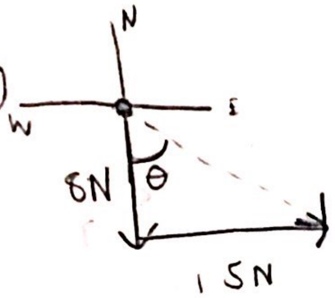
Review for Quest 12.1 - 12.4

1)



$$\|\vec{F_3}\| = \sqrt{8^2 + 15^2}$$

$$= 17 \text{ N}$$

$$\tan\theta = \frac{15}{8}$$

$$\theta = 61.93^\circ$$

Direction from North

$90^\circ + (90 - 61.93) = 118.07^\circ$

Solution: Magnitude F_3 = 17N

Direction clockwise from North = 118.07°

or S 61.9° E

2) a) $\vec{AB} = -\vec{u}$

b) $\vec{CA} = \vec{u} - \vec{v}$

c) $\vec{CT} = \frac{3}{5}(\vec{CA})$

$= \frac{3}{5}(\vec{u} - \vec{v})$

$\boxed{\vec{CT} = \frac{3}{5}\vec{u} - \frac{3}{5}\vec{v}}$

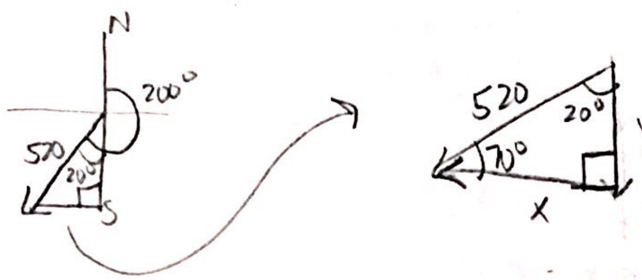
d) $\vec{BT} = \vec{BA} + \vec{AT}$

$= \vec{u} + \frac{2}{5}(\vec{AC})$

$= \vec{u} + \frac{2}{5}(\vec{v} - \vec{u})$

$\boxed{\vec{BT} = \frac{3}{5}\vec{u} + \frac{2}{5}\vec{v}}$

3)



$$\cos 20^\circ = \frac{y}{520}$$

$y = -488.64$ mi/h

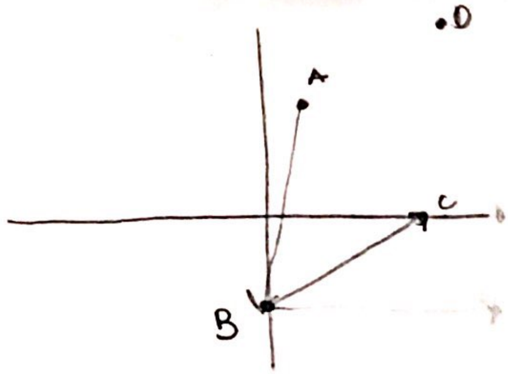
$$\sin 20^\circ = \frac{x}{520}$$

$x = -177.85$ mi/h

Solution:

Plane's velocity vector 177.81 mi/h west, 488.64 mi/h south.

④



$\vec{AB} = (-2, -9)$

$\vec{AB} = \vec{DC}$ - definition of parallelogram

$\vec{DC} = (-2, -9) : (7 - x_D, 0 - y_D)$

$-2 = 7 - x_D \quad -9 = 0 - y_D$

$9 = x_D \quad 9 = y_D$

$D(9, 9)$

Check: $\vec{BC} = \vec{AD}$ definition of parallelogram

$\vec{BC} = (7-0, 0-(-5))$
$= (7, 5)$

$\vec{AD} = (9-2, 9-4)$
$= (7, 5)$

$\vec{BC} = \vec{AD}$ ✓

Point $D(9,9)$

⑤ a) velocity $= (2, -3)$

$(x, y) = (1, 3) + t(2, -3)$

Starting point at $t=0$ — velocity

Speed $= \|\text{velocity}\| = \sqrt{2^2 + (-3)^2}$
$= \sqrt{4+9}$
Speed $= \sqrt{13}$

b) $x = 1 + 2t$
$y = 3 - 3t$

c) Substitute x and y from part b into equation of curve.

$3 - 3t = 2(1+2t)^2 - 6(1+2t)$
$3 - 3t = 2(4t^2 + 4t + 1) - 6 - 12t$
$3 - 3t = 8t^2 + 8t + 2 - 6 - 12t$
$0 = 8t^2 - t - 7$

$t = \dfrac{1 \pm \sqrt{1^2 - 4(8)(-7)}}{2(8)}$

$t = 1, -0.875$

Position: $(3, 0)$ and $\left(\frac{-3}{4}, 5\frac{5}{8}\right)$

⑥

a) $\overrightarrow{AB} = (1-(-2),\ 1-4)$

$\overrightarrow{AB} = (3, -3)$

* $\overrightarrow{BA}$ is in opposite direction $(-3, 3)$

b) $\overrightarrow{AP} = \frac{2}{3}\overrightarrow{AB}$

$= \frac{2}{3}(3,-3) = (2,-2)$

$\overrightarrow{AP} = (2,-2) = (x_P - x_A,\ y_P - y_A)$

$2 = x_P - x_A$ $\qquad -2 = y_P - y_A$

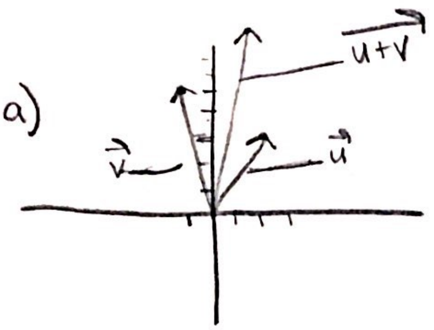
$2 = x_P - (-2)$ $\qquad -2 = y_P - 4$

$2 = x_P + 2$ $\qquad 2 = y_P$

$0 = x_P$

$P(0,2)$

⑦ a)



$\overrightarrow{u+v} = (2,3) + (-1,5)$

$= (2+-1,\ 3+5)$

$\boxed{\overrightarrow{u+v} = (1, 8)}$

b) $\|\vec{u} + 2\vec{v}\| = \|(2,3) + 2(-1,5)\|$

$= \|(2,3) + (-2,10)\|$

$= \|(2+-2,\ 3+10)\|$

$= \|(0, 13\|$

$= \sqrt{13^2}$

$\boxed{\|\vec{u} + 2\vec{v}\| = 13}$

c) $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} = \frac{2\cdot -1 + 3\cdot 5}{\sqrt{13}\cdot\sqrt{26}} = \frac{13}{\sqrt{338}} = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = 45°$

8

a) $m(-2,4) = (3, k)$

$k = -6$

$(-2m, 4m) = (3, k)$

$-2m = 3$ $\quad$ $4m = k$

$m = \frac{-3}{2}$ $\quad$ $4\left(\frac{-3}{2}\right) = k$

$-6 = k$

b) $(-2,4) \cdot (3, k) = 0$

$-2 \cdot 3 + 4 \cdot k = 0$

$-6 + 4k = 0$

$k = \frac{3}{2}$

c) $(x, y) = (8, 9) + t(-2, 4)$

$x = 8 - 2t$

$y = 9 + 4t$