Name: ______________________ 614 Chapter 4 Functions Review

1. The radius r, in inches , of a spherical balloon is related to the volume, V, by $r(V)=\sqrt[3]{\frac{3V}{4\pi}}$. Air is pumped into the balloon so the volume after t seconds is given by V(t) = 10+20t.

a.) Find the composite function r(V(t))

$r(V(t)) = \sqrt[3]{\frac{3(10+20t)}{4\pi}} \Rightarrow \sqrt[3]{\frac{30+60t}{4\pi}}$

b.) Find the time when the radius reaches 10 inches.

$10 = \sqrt[3]{\frac{30+60t}{4\pi}}$

$10^3 = \frac{30+60t}{4\pi}$ $4000\pi = 30+60t$

$t = 208$ sec.

2. Describe the transformation of each function from its parent.

a.) $f(x) = 4(x+1)^2 - 5$

left 1 , ↓ 5 , vertical stretch factor of 4

b.) $h(x) = -2|x-4| + 3$

right 4 reflects over x-axis vertical stretch factor of 2

↑ 3

c.) $g(x) = \frac{1}{2}x^3$

Vertical shrink factor of 1/3

4. If $f(x)=\frac{x}{2+x}$ and $g(x)=\frac{2x}{1-x}$ find

a.) g(f(x))

$g\left(\frac{x}{2+x}\right) = \frac{2\left(\frac{x}{2+x}\right)}{1-\left(\frac{x}{2+x}\right)} = \frac{\frac{2x}{2+x}}{\frac{2+x-x}{2+x}} = \frac{2x}{2+x} \cdot \frac{\cancel{2+x}}{2} = \frac{2\cancel{x}}{2} = x$

b.) f(g(x))

$f\left(\frac{2x}{1-x}\right) = \frac{\frac{2x}{1-x}}{2+\frac{2x}{1-x}} = \frac{\frac{2x}{1-x}}{\frac{2(1-x)+2x}{1-x}} = \frac{2x}{\cancel{1-x}} \cdot \frac{\cancel{1-x}}{2} = x$

c.) What does this tell you about the relation between f(x) and g(x)? Be specific.

They are inverses of one another

Find the domain for each function.

$f(x) = \frac{x}{\ldots}$

c.) $h(x)=\dfrac{2x-1}{\sqrt{3x+1}}$

$\{x \mid x \neq 2\}$

$\{x \mid x \neq \pm 3\}$

$3x+1 > 0$
$3x > -1$
$x > -1/3$

$\{x \mid x > -1/3\}$

note $3x+1 \neq 0$ since it is in the denominator of the function

6. If f(x) = 3x −5, find $\dfrac{f(x)-f(a)}{x-a}$

$$\frac{3x-5-(3a-5)}{x-a} = \frac{3x-3a}{x-a} = \frac{3(x-a)}{x-a} = 3$$

7. Let $f(x)=3x-1$ $g(x)=x^2-2$ $h(x)=\sqrt{10-2x}$ $k(x)=\dfrac{1}{3x-4}$, find

a.) f(7)

$3(7)-1 = 20$

b.) k(−4)

$\dfrac{1}{3(-4)-4} = \dfrac{1}{-16}$

c.) $f^{-1}(x)$

$y = 3x-1$
$x = 3y-1$
$x+1 = 3y$
$\frac{x+1}{3} = f^{-1}(x)$

d.) (h+g)(7)

$h(7)+g(7)$
$\sqrt{10-2(7)} + 7^2-2$
$\sqrt{10-14} + 49-2$
$= \sqrt{-4} + 47$
$= 2i + 47$

e.) $(g \circ f)(x)$

$g(3x-1) = (3x-1)^2 - 2$
$= 9x^2-6x+1-2$
$= 9x^2-6x-1$

f.) Domain for h(x)

$10-2x \geq 0$
$10 \geq 2x$
$5 \geq x$

$\{x \mid x \leq 5\}$

g.) $g(g^{-1}(x))$

$= x$

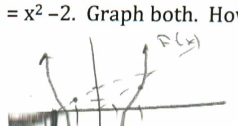
(inverses)

h.) f(k(x))

$f\left(\frac{1}{3x-4}\right) = 3\left(\frac{1}{3x-4}\right)-1$
$= \frac{3}{3x-4} - 1$
$= \frac{3-(3x-4)}{3x-4}$
$= \frac{-3x+7}{3x-4}$

8. Find the inverse of f(x) = x^2 −2. Graph both. How are they related?

$y = x^2-2$
$x = y^2-2$



9. Find the inverse of $k(x) = \frac{x-4}{x-2}$. Note any domain restrictions.

$x \neq 2$

$y = \frac{x-4}{x-2}$

$x = \frac{y-4}{y-2}$

$x(y-2) = y-4$

$xy - 2x = y - 4$

$xy - y = 2x - 4$

$y(x-1) = 2x-4$

$y = \frac{2x-4}{x-1}$ $x \neq 1$

10. What makes a function 1-1? Be very specific in your explanation.

A 1-1 function is

11. a.) Graph the relation. $f(x) = \begin{cases} x+5 & x<-2 \\ x^2+2x+3 & x \geq -2 \end{cases}$

b.) Is it a function? Yes

c.) f(3) = use piece #2

$3^2 + 2(3) + 3 = 9 + 6 + 3 = 18$

d.) f(−4) = use piece #1

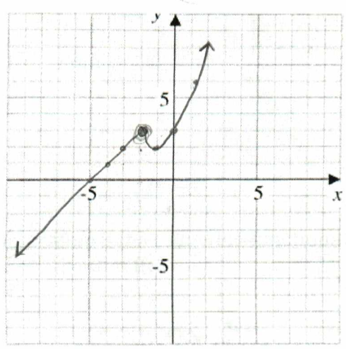
$-4 + 5 = 1$

e.) f(−2) use piece #2

$(-2)^2 + 2(-2) + 3 = 3$

f.) Domain? $\mathbb{R}$

g.) Range? $\mathbb{R}$



* note $-2 + 5 = 3$

$(-2)^2 + 2(-2) + 3 = 3$

Since $f(-2) = 3$ on both functions the point is solid including $(-2, 3)$

9. Find the inverse of $k(x) = \dfrac{x-4}{x-2}$. Note any domain restrictions. $x \neq 2$

$y = \dfrac{x-4}{x-2}$

$x = \dfrac{y-4}{y-2}$

$x(y-2) = y-4$

$xy - 2x = y - 4$

$xy - y = 2x - 4$

$y(x-1) = 2x - 4$

$y = \dfrac{2x-4}{x-1}$ $x \neq 1$

10. What makes a function 1-1? Be very specific in your explanation.

A 1-1 function is if every input is assigned exactly one unique output. Both the function and it's inverse are functions. The original graph passes both the horizontal and vertical line test.

11. a.) Graph the relation. $f(x) = \begin{cases} x+5 & x < -2 \\ x^2+2x+3 & x \geq -2 \end{cases}$

b.) Is it a function? Yes

c.) f(3) = use piece #2

$3^2 + 2(3) + 3 = 9 + 6 + 3 = 18$

d.) f(−4) = use piece #1

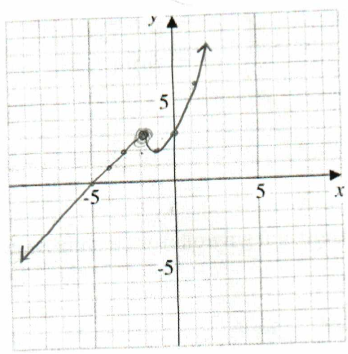
$-4 + 5 = 1$

e.) f(−2) use piece # 2

$(-2)^2 + 2(-2) + 3 = 3$

f.) Domain? $\mathbb{R}$

g.) Range? $\mathbb{R}$



* note

$-2 + 5 = 3$

$(-2)^2 + 2(-2) + 3 = 3$

Since $f(-2) = 3$ on both functions the point is solid including $(-2, 3)$

Use the graph of f(x)

a.) State the domain and range of f.

D: $\{x \mid -4 \le x \le 8\}$

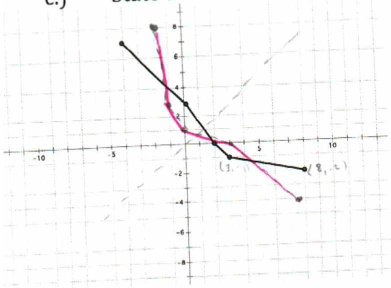
R: $\{y \mid -2 \le y \le 7\}$

b.) Sketch the inverse

c.) State the domain and range of the inverse.

$D_{inverse}$ $\{x \mid -2 \le x \le 7\}$

$R_{inverse}$ $\{y \mid -4 \le y \le 8\}$



3. If f and g are inverse functions, f(−2) = 3 and f(4) = −2, find g(−2).

$g(-2) = 4$

4. Use the table to find the indicated quantitites.

X	0	1	2	3	4	5	6	7	8	9
F(x)	8	0	7	4	2	6	5	3	9	1

a.) find f(1) $= 0$

b.) Solve f(x)=3 $= 7$

c.) Find $f^{-1}(0)$ = 1

d.) Solve $f^{-1}(x) = 7$ $= 3$

15. Determine if the following functions are even or odd. Explain.

a.) |x+3| = f(x)

b.) g(x) = 1/5x

even - symmetry over the y-axis $(x,y) \to (-x,y)$

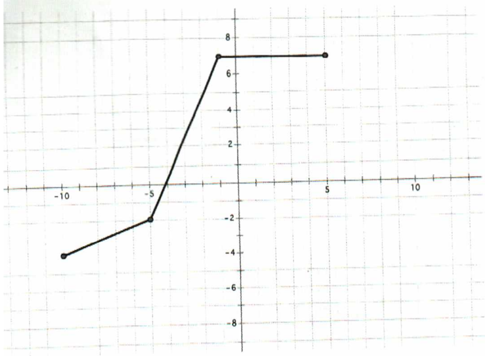
odd - point symmetry 180° around origin $(x,y) \to (-x,-y)$

a.) $|-x+3| \ne |x+3|$ not even nor odd

b.) $-y = \frac{1}{5}(-x)$

$y = \frac{1}{5}x$ odd

6. Use the function f(x) sketched below



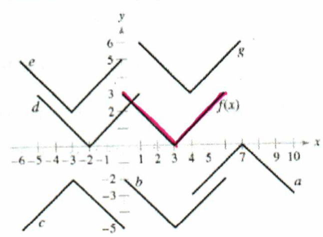
a.) Graph $-f(x)$ reflects over x-axis y values are opposites

b.) Graph $f(x+3)$ moves left 3

c.) Graph $|f(x)|$ all outputs are +

d.) Graph $1/2f(x) + 2$ ↑ 2 y values · by ½

Use the graph of $y = f(x)$ to match the function with its graph.



1) $g(x) = f(x+5)$ d	2) $g(x) = -f(x-4)$ a	3) $g(x) = f(x-1)+3$ g	4) $g(x) = f(x)-5$ b

3. An automobile race track is to be constructed in the shape of two parallel section connected by two semicircles. The track (one round trip) is to be exactly 2.5 miles in length.

a. Write an equation for the area of the shaded rectangular region as a function of the radius r.
b. Draw a complete graph of the function in part a.
c. What values of r make sense in this problem situation?
d. Use a graph to determine the value of the radius that produces the maximum rectangular area. What is the maximum rectangular area?

$2\pi r + 2l = 2.5$

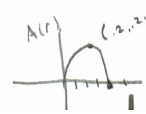
$2l = 2.5 - 2\pi r$

$l = \frac{2.5 - 2\pi r}{2}$

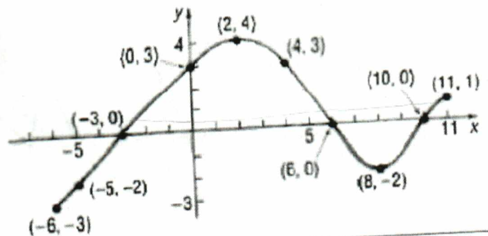
$A(r) = 2r\left(\frac{2.5 - 2\pi r}{2}\right)$

c) Domain

$2\pi r = 2.5$



19. Use the graph of the given function f to answer Questions 1 – 13.



1	Find $f(0)$ and $f(-6)$.	$f(0) = 3$ $f(-6) = -3$
2	Find $f(6)$ and $f(11)$	$f(6) = 0$ $f(11) = 1$
3	Is $f(3)$ positive or negative? Explain your reasoning.	+, above x-axis
4	Is $f(-4)$ positive or negative? Explain your reasoning.	− below x-axis
5	For what numbers x is $f(x)=0$?	$x = -3, 6, 10$
6	For what numbers x is $f(x)>0$?	$-3 < x < 5$ and $10 < x \le 11$
7	What is the domain of f?	$\{x \mid -6 \le x \le 11\}$
8	What is the range of f?	$\{y \mid -3 \le y \le 4\}$
9	What are the x-intercepts?	−3, 6, 10
10	What are the y-intercepts?	3
11	For how many values of x does $f(x)=\frac{1}{2}$?	3
12	How often does the line $x=5$ intersect the graph of f?	1
13	For what values of x does $f(x)=3$?	0

20. a.) Find the axis of symmetry for $f(x)=3x^2-8x+7$.

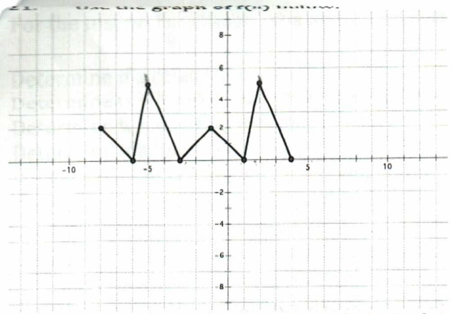
$V_x = \frac{-b}{2a}$ $\frac{8}{6} = \frac{4}{3}$ $x = \frac{4}{3}$

b.) Find the point of symmetry for $p(x)=2x^3+12x^2+9$

$P_{sx} = \frac{-b}{3a}$ $\frac{-12}{2(3)} = \frac{-12}{6} = -2$

$p(-2) = 2(-2)^3 + 12(-2)^2 + 9$

$= 41$



a.) Is the function periodic? Explain.

yes - repeats itself.

b.) If so, what is the fundamental period?

$|2 - -5| = 7$

c.) What is the amplitude of the function?

$A = \frac{max - min}{2} = \frac{5-0}{2} = 2.5$

d.) Find f(32)

$\frac{32}{7}$

$32 = 28 + 4$ $\quad$ $R(4)$ $\quad$ $f(4) = 0$

22. Determine if the following equations are symmetric over (a) the x-axis (b) the y-axis (c) (0,0) (d) over y = x

a.) $y = |x| - 9$

x-axis $\quad (x, y) \to (x, -y)$

y-axis $\quad (x, y) \to (-x, y)$

(0,0) Rot $\quad (x, y) \to (-x, -y)$

$y = x \quad (x, y) \to (y, x)$

1.) $-y = |x| - 9$ no

2.) $y = |-x| - 9$ yes

3.) $-y = |-x| - 9$ NO

b.) $x^2 + y^2 = 12$

① $x^2 + (-y)^2 = 12$ yes

② $(-x)^2 + (y)^2 = 12$ yes

③ $(-x)^2 + (-y)^2 = 12$ yes

④ $(y)^2 + (x)^2 = 12$ yes

* this is a circle $r = \sqrt{12}$

$C(0,0)$