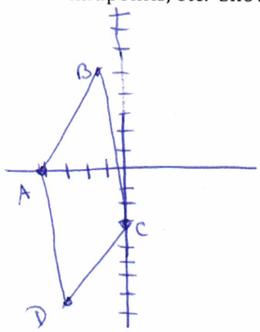
Quadrilaterals on the Coordinate Plane

1. Show that the points A(-4, 0), B(-1, 5), C(0, -3), and D(-3, -8) form parallelogram ABCD and that ABCD is not a rectangle. A figure is not sufficient justification. You must use slopes, distances, midpoints, etc. Show calculations.



To prove parallelogram opposite sides must be ∥ (same slope)

$m\overline{AB} = \frac{5}{3}$

$m\overline{DC} = \frac{-5}{-3} = \frac{5}{3}$) = $\overline{AB} \parallel \overline{DC}$ both have slopes $\frac{5}{3}$

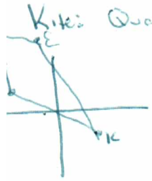
$m\overline{AD} = \frac{8}{-1}$

$m\overline{BC} = \frac{8}{-1}$) = $\overline{AD} \parallel \overline{BC}$ both have slopes -8

∴ ABCD is a ▱

Since adjacent sides do not have slopes that are opposite reciprocals the quad does not have any ⊥ sides (right ∡) ∴ ABCD is a parallelogram

2. Show that the points K(2, -2), I(-4, 1), T(-4, 4), and E(-1, 4) form a kite. A figure is not sufficient justification. You must use slopes, distances, midpoints, etc. Show calculations.



Kite: Quad prove adj sides are ≅, diagonals are ⊥

Prove $IT = TE$ $\quad IT = \sqrt{(-4--4)^2 + (1-4)^2} = \sqrt{9} = 3$ $\quad TE = \sqrt{(4--1)^2 + (4-4)^2} = \sqrt{9} = 3$

$KI = KE$ $\quad KI = \sqrt{(2--4)^2 + (-2-1)^2} = \sqrt{45} = 3\sqrt{5}$ $\quad KE = \sqrt{(2--1)^2 + (-2-4)^2} = \sqrt{45} = 3\sqrt{5}$

2 pair of adjacent ≅ sides

$m\overline{KT} = \frac{-2-4}{2--4} = \frac{-6}{6} = -1$ $\quad m\overline{IE} = \frac{4-1}{-1--4} = \frac{3}{3} = 1$ $\quad \overline{KT} \perp \overline{IE}$

Since $m\overline{KT} \cdot m\overline{IE} = -1$

diagonals of quad are ⊥ ∴ KITE forms a kite

3. Given the points A(-2, -1), and B(4, -3), determine coordinates for points C and D so that they satisfy the conditions in each of the problems given below. **Justify your answer using side lengths or diagonals**. A figure is not sufficient justification. You must use slopes, distances, midpoints, etc. Show calculations. There is not sufficient space below to do these problems, so use a separate sheet of paper.

   a. ABCD is a rectangle but not a square. ▱ with one right ∡

   b. ABCD is a parallelogram but not a rectangle. ▱ no right ∡s

   c. ABCD is a trapezoid but not a parallelogram nor an isosceles trapezoid.
   one pair of parallel sides no ≅

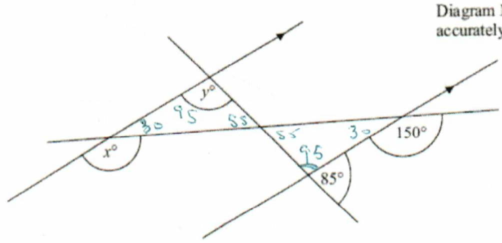
   d. ABCD is an isosceles trapezoid but not a parallelogram.
   trapezoid one pair of ∥ sides, 2 ≅ legs

   d. ACBD is a square (note the order of the letters on this one!!).

arallel Lines Cut by a Transversal

1. Solve for x and y. Identify, by name, any special angles and relationships used.

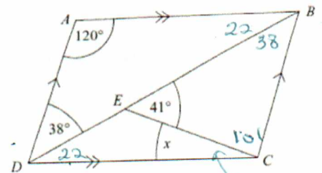
Diagram **NOT** accurately drawn



1) Solve for supplementary ∡
2.) used parallel lines & alternate interior ∡s to find y
3.) sum of ∡s of Δ is 180
4.) vertical ∡s are ≅
5) sum of ∡s of Δ is 180
6.) x is supplementary
x = 150
or x is corresponding to 150°

2.

Diagram **NOT** accurately drawn



120 - 101 = 19°

ABCD is a parallelogram.

Angle *ADB* = 38°.
Angle *BEC* = 41°.
Angle *DAB* = 120°.

Calculate the size of angle *x*.
You must give reasons for your answer.

1.) Alternate interior ∡ADB & ∠DBC ≅
2) sum of ∡s in Δ = 180 - solve for ∡ABD = 22°
3) opp. ∡s in ▱ are ≅
∠BCD = 120°
4.) sum of ∡s in Δ add to 180
∠BCE = 101°
5.) 120 - 101 = 19° (steps 3 & 4)

3.

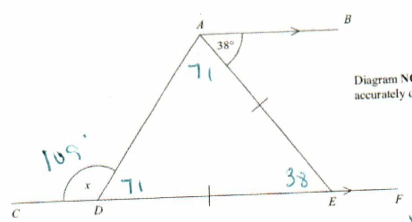


Diagram **NOT** accurately drawn

$\frac{180-38}{2} = 71$

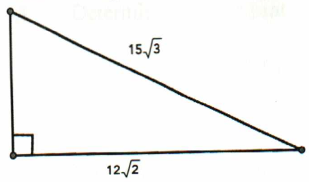
CDEF is a straight line.
AB is parallel to *CF*.
DE = *AE*.

Work out the size of the angle marked *x*.
You must give reasons for your answer.

1.) ∠AED is alternate interior to ∥ lines & 38° ∡
2.) ΔAED is isosceles with base ∡ EAD & ∠EDA
... 109°

611 Review Unit 5 TEST

Find the length of the missing side. Your answer should be expressed in simplest radical form.



$(\text{leg})^2 + (\text{leg})^2 = (\text{hypotenuse})^2$

$(15\sqrt{3})^2 + (12\sqrt{2})^2 = h^2$

$(15 \cdot 15 \cdot \sqrt{3} \cdot \sqrt{3}) + (12 \cdot 12 \cdot \sqrt{2} \cdot \sqrt{2}) = h^2$

$225 \cdot 3 + 144 \cdot 2 = h^2$

$675 + 288 = h^2$

$963 = h^2$ $\quad h = \sqrt{963} = 3\sqrt{107}$

5. Triangle DAN has coordinates D (2, 10), A (6, 4) and N(12,8). Using coordinate geometry, decide the most descriptive name for the triangle.

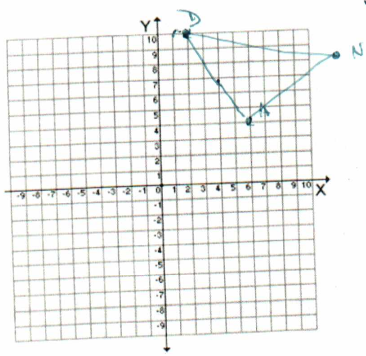
using slope formula

$m\overline{DA} = \frac{10-4}{2-6} = \frac{6}{-4} = -\frac{3}{2}$

$m\overline{AN} = \frac{8-4}{12-6} = \frac{4}{6} = \frac{2}{3}$

$-\frac{3}{2} \cdot \frac{2}{3} = -1$ $\therefore$ $\overline{DA} \perp \overline{AN}$

& ∠DAN is a right ∡



using distance formula

$DA = \sqrt{(2-6)^2 + (10-4)^2} = \sqrt{4^2 + 6^2} = \sqrt{16+36} = \sqrt{52}$

$AN = \sqrt{(6-12)^2 + (4-8)^2} = \sqrt{6^2 + 4^2} = \sqrt{52}$

$DN = \sqrt{(12-2)^2 + (8-10)^2} = \sqrt{100+4} = \sqrt{104}$

Since $\overline{DA} \cong \overline{AN}$ △DAN is isosceles (a △ with 2 ≅ sides)

∴ △DAN is an isosceles right △

* Now you could show that the Pythagorean Theorem is satisfied to prove a right △

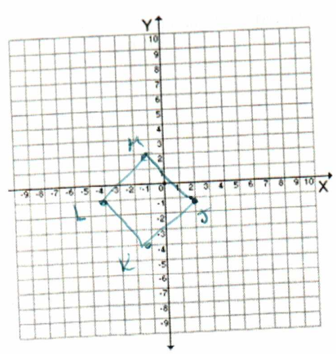
6. Prove that a quadrilateral with vertices J(2,-1), K(-1,-4), L(-4,-1) and M(-1, 2) is a square.

Square is a ▱ with one right ∡ & 4 ≅ sides

to show ▱ use slope formula

$m\overline{MJ} = \frac{2--1}{-1-2} = \frac{3}{-3} = -1$ $\quad m\overline{LK} = \frac{-4--1}{-1--4} = \frac{-3}{3} = -1$

$m\overline{ML} = \frac{2--1}{-1--4} = \frac{3}{3} = 1$ $\quad m\overline{JK} = \frac{-4--1}{-1-2} = \frac{-3}{-3} = 1$



Since opposite sides of quad have same slope

$m\overline{MJ} = m\overline{LK}$ $\quad m\overline{ML} = m\overline{JK}$

the segments are ∥ making quad MLKJ a parallelogram

Since $-1 \cdot 1 = -1$ $\overline{MJ} \perp \overline{ML}$ forming a right ∡

MLKJ is a rectangle

Since adjacent sides are = the figure is a square

$\sqrt{9+9} = \sqrt{18}$

611 Review Unit 5 TEST — Name: ____________

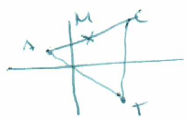
A triangle has vertices C (3, 5) A(−2,1) and T(4, −3).

a.) Determine the midpoint of $\overline{CA}$.

midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$M\left(\frac{3+-2}{2}, \frac{5+1}{2}\right) = \left(\frac{1}{2}, 3\right)$

b.) Determine the equation of the median for ΔCAT from point T.



median – segment with endpoints at vertex & midpoint of opp side

use point $M\left(\frac{1}{2}, 3\right)$ point T (4, −3)

$m = \left(\frac{3--3}{\frac{1}{2}-4}\right) = \frac{6}{-3\frac{1}{2}} = \frac{-12}{7}$

$y+3 = \frac{-12}{7}(x-4)$

or $y-3 = -\frac{12}{7}\left(x-\frac{1}{2}\right)$

c.) Find the equation of the line that is parallel to $\overline{CT}$ containing point A.

$m\overline{CT} = \frac{5--3}{3-4} = \frac{8}{-1} = -8$

parallel lines have same slope

$y-1 = -8(x+2)$

$y = -8x - 15$

d.) Find the equation of the perpendicular bisector of $\overline{AT}$.

midpoint $\overline{AT}\left(\frac{-2+4}{2}, \frac{1+-3}{2}\right) = (1, -1)$

slope $\overline{AT}$ $\left(\frac{-3-1}{4--2}\right) = \frac{-4}{6} = \frac{-2}{3}$ $m_{\perp} = \frac{3}{2}$

$y+1 = \frac{3}{2}(x-1)$

$y = \frac{3}{2}x - 2.5$

8. Find the coordinates of the point that is 1/3 of the way from (–8, 4) and (4, 12).

3 parts −8 → 4 12 horizontal units $\frac{12}{3} = 4$

4 → 12 8 vertical units $\frac{8}{3} = 2\frac{2}{3}$

$(-8+4, 4+2^2/_3)$

(1) $(-4, 6^2/_3)$

or $(4-4, 12-2^2/_3)$

$(0, 9\frac{1}{3})$

Note: There is only ONE two column proof on the test.

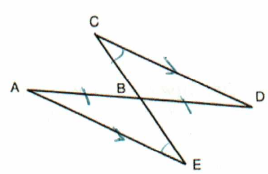
9.

Given: $\overline{AE} \parallel \overline{CD}$

$\overline{AB} \cong \overline{BD}$

Prove: $\Delta ABE \cong \Delta DBC$

Write a flow proof or a 2-column proof.

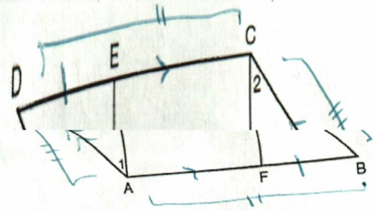


Statements	Reasons
1. $\overline{AE} \parallel \overline{CD}$, $\overline{AB} \cong \overline{BD}$	1. given
2. $\angle C \cong \angle E$	2. given 2 ∥ lines cut by a transversal alternate interior ∠ are ≅
3. $\angle CBD \cong \angle ABE$	3. vertical ∠s are ≅
4.	4. AAS

611 Review Unit 5 TEST

Name: ____________________

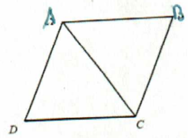
parallelogram



Prove: a) $\Delta DEA \cong \Delta BFC$
b) $\angle 1 \cong \angle 2$

Statements	Reasons
1. $\square ABCD$, $\overline{DE} \cong \overline{FB}$	1. given
2. $\angle D \cong \angle B$	2. opp ∡s in $\square$ are ≅
3. $\overline{CB} \cong \overline{DA}$	3. opp sides of $\square$ are ≅
4. $\Delta DEA \cong \Delta BFC$	4. SAS
5. $\angle 1 \cong \angle 2$ ($\angle DAE \cong \angle BCF$)	5. CPCTC

11.



Given: $\square ABCD$

Prove: $\Delta DAC \cong \Delta BCA$

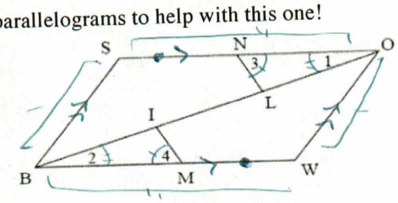
(At most 6 steps! You may not need all 6!!!)

Statements	Reasons
1. $\square ABCD$	1. given
2. $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$	2. opp sides of $\square$ are ≅
3. $\angle B \cong \angle D$	3. opp ∡ in $\square$ are ≅
4. $\Delta DAC \cong \Delta BCA$	4. SAS

12. Use your knowledge of parallelograms to help with this one!

Given: *WOSB* is a parallelogram
$\measuredangle 3 \cong \measuredangle 4$
$\overline{MW} \cong \overline{SN}$

Prove: $\overline{IM} \cong \overline{LN}$



Statements	Reasons
1. WOSB is $\square$, $\angle 3 \cong \angle 4$, $\overline{MW} \cong \overline{SN}$	1. given
1.5 $\overline{SO} \parallel \overline{BW}$	1.5 opp sides of $\square$ are ∥
2. $\angle 1 \cong \angle 2$	2. given 2 ∥ lines cut by transversal ($\overline{SO} \parallel \overline{BW}$, transversal $\overline{BO}$) alternate interior ∡ are ≅.
3. $\overline{BW} \cong \overline{SO}$	3. opp sides of $\square$ are ≅