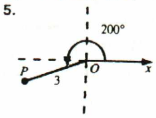
Chapter 11:
Polar Coordinates and Complex Numbers

Sections 11-1, 11-2, 11-3, and 11-4

1. C **2.** B **3.** D **4.** A **5.**



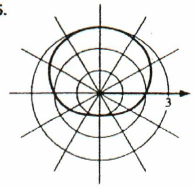
Answers will vary, sample answers: $(3, -160°)$, $(-3, 20°)$.

6. Answers will vary, sample answer: $\left(4\sqrt{2}, \frac{3p}{4}\right)$. **7.** $(\sqrt{3}, 1)$

8. 17 cis 298° **9.** about $1.71 + 4.70i$ **10.** $2 \text{ cis } \frac{3p}{4}; -\sqrt{2} + i\sqrt{2}$

11. Answers will vary, a sample answer is given. Manoj's method is valid. Since $a = r\cos\theta$ and $b = r\sin\theta$, $\cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+b^2}}$ and $\sin\theta = \frac{b}{r} = \frac{b}{\sqrt{a^2+b^2}}$. His method specifies values for both $\sin\theta$ and $\cos\theta$; this completely specifies θ. **12. a.** $\sqrt{2}$ cis 45° **b.** $z^{-1} = \frac{\sqrt{2}}{2} \text{ cis } 315°$, $z^0 = 1$, $z^2 = 2 \text{ cis } 90°$ **c.** $(1 + i)(1 + i) = 1 + 2i + (-1) = 2i$ and $2 \text{ cis } 90° = 2(0 + i) = 2i$; they are equal. **13. a.** $\sqrt{3} + i, -1 + i\sqrt{3}, -\sqrt{3} - i, 1 - i\sqrt{3}$ **b.** $(\sqrt{3} + i)^4 = (2 + 2i\sqrt{3})^2 = 4 + 8i\sqrt{3} + 12i^2 = -8 + 8i\sqrt{3}$; yes **14.** $(9.8, 0.4)$

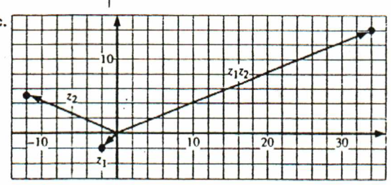
15.



$(x^2 + y^2 - y)^2 = 4x^2 + 4y^2$

16. a. $34 + 14i$ **b.** $z_1 = (2\sqrt{2}, 225°)$; $z_2 = (13, 157.4°)$; $z_1z_2 = (26\sqrt{2}, 382.4°)$, or $(26\sqrt{2}, 22.4°) = 34 + 14i$

c.

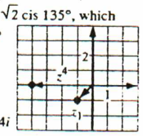


17. This rotates the graph of z 90° counterclockwise; let $z = r \text{ cis } \theta$, since $i = \text{cis } 90°$, $iz = (\text{cis } 90°)(r \text{ cis } \theta) = r \text{ cis } (90° + \theta)$; thus, iz has the same absolute value as z but its graph is obtained by rotating the graph of z 90° counterclockwise. **18.** Answers will vary, a sample answer is given. A radar screen usually has a circular picture of an area with the center corresponding to the location of the radar unit. The screen has a compass scale around the edge for direction readings. Radar uses the distance and direction of an object to locate it, just like the polar coordinate system does. **19. a.** $-\frac{1}{2} - \frac{1}{2}i$ **b.** $-\frac{1}{2} - \frac{1}{2}i$ **20.** cis 54°, cis 126°, cis 198°, cis 270°, cis 342°

Chapter 11 Review

Quick Check

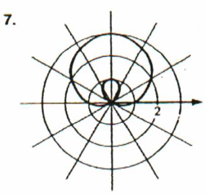
1. $(4\sqrt{2}, -4\sqrt{2})$ **2.** Make a table of values to find values of r for known values of θ (multiples of 30° and 45°). Continue until the points begin to repeat. Plot the points to obtain the graph. **3.** $2i = 2 \text{ cis } 270°$; $-1 - i = \sqrt{2} \text{ cis } 225°$; $-2i(-1 - i) = -2 + 2i$; $(2 \text{ cis } 270°)(\sqrt{2} \text{ cis } 225°) = 2\sqrt{2} \text{ cis } 495° = 2\sqrt{2} \text{ cis } 135°$, which agrees with $-2 + 2i$. **4.** $z^4 = (\sqrt{2})^4 \text{ cis } 900° = 4 \text{ cis } 180° = -4$ **5.** $4i = 4 \text{ cis } 90°$; $2 \text{ cis } 45° = \sqrt{2} + i\sqrt{2}$ and $2 \text{ cis } 225° = -\sqrt{2} - i\sqrt{2}$; $(\sqrt{2} + i\sqrt{2})^2 = 2 + 4i + 2i^2 = 4i$ and $(-\sqrt{2} - i\sqrt{2})^2 = 2 + 4i + 2i^2 = 4i$



Practice Test

1. $(2, 90°)$ **2.** $(6\sqrt{2}, -45°)$ **3.** $(5, 143.1°)$ **4.** $(-2\sqrt{2}, 2\sqrt{2})$ **5.** $(-4\sqrt{3}, -4)$ **6.** $(6.43, -7.66)$

7.



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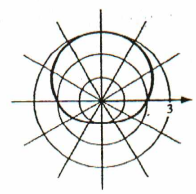

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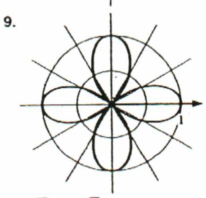

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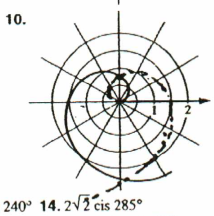
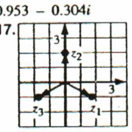


11. $\sqrt{2}$ **12.** $\sqrt{2}$ cis 45° **13.** 2 cis 240° **14.** $2\sqrt{2}$ cis 285° **15.** $-0.988 - 0.151i$ **16.** $-0.213 + 0.977i, -0.740 - 0.673i, 0.953 - 0.304i$

17.



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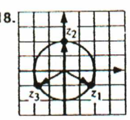


19. z_1, z_2, and z_3 all lie on the circle $|z| = 2$ because $|z_1| = |z_2| = |z_3| = 2$.

3. Find the square roots of $2-2i$

$r=\sqrt{4+4}=\sqrt{8}$

$\theta=\tan^{-1}-1=-45°$ 315

$z^2=\sqrt{8}cis(-45°)$ $\sqrt{8}$ cis 315

$(rcis\theta)^2=\sqrt{8}cis(-45°)$ $r^2=\sqrt{8}$ $2\theta=315+360k$

$\theta=157.5+180k$

$r^2cis2\theta=\sqrt{8}cis(-45°)$

$r^2=8^{1/2}\quad 2\theta=-45°+360k$ $8^{1/4}$ cis 157.5

$r=8^{1/4}\quad \theta=-22.5°+180k$ $8^{1/4}$ cis 337.5°

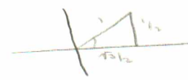
$k=0\quad z_1=8^{1/4}cis(-22.5°)=1.554-.646i$

$k=1\quad z_2=8^{1/4}cis(157.5°)=-1.554+.646i$

Practice Problems
Polar and Complex Numbers

Name ____________________

1. If $z = \frac{\sqrt{3}}{2} + \frac{1}{2}i$, express z in polar form.
   Calculate z^2 and z^3 by using De Moivre's theorem.



1 cis 30

$z^2 = $ 1 cis 60 $z^3 = $ 1 cis 90

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2. Compute $(1 - i\sqrt{3})^5$. Express your answer in polar and rectangular form.

2 cis 300

$z^5 = 2^5$ cis 300 · 5

= 32 cis 1500 = 32 cis 60 = 32(cos 60 + i sin 60) = 16 + 16√3 i

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3. Find all three cube roots of 8i. Call them z_1, z_2, and z_3.
   Express your answers in rectangular form.

8i = 8 cis 90

$z^3 = r^3$ cis nθ

8 cis 90 = r^3 cis 3θ

$r^3 = 8$

r = 2

z_1 = 2 cis 30 √3 + 1i

z_2 = 2 cis 150 = −√3 + 1i

z_3 = 2 cis 270 = −2i

2 cis (90/3 + 0)

30

2 cis (90/3 + 360/3) = 2 cis 150

2 cis (90/3 + 2·360/3) = 2 cis 270

Find the sum: $z_1 + z_2 + z_3$ = 0

Find the product: $z_1 \cdot z_2 \cdot z_3$ = 8i

(√3 + i)(−√3 + i)
(−3 − 1) −2i = −4 · −2i = 8i

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4. Find all three cube roots of -8. Call them z_1, z_2, and z_3.
   Express your answers in rectangular form.

−8 = 8 cis 180

$z^3 = r^3$ cis 3θ

8 cis 180 = r^3 cis 3θ

r = 2

3θ = 180

θ = 60

z_1 = 2 cis 60 1 + √3 i

z_2 = 2 cis 180 −2

z_3 = 2 cis 300 1 − √3 i

2 cis (180/3 + 360/3)

2 cis 180

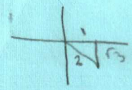
2 cis (180/3 + 720/3)

2 cis 300

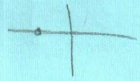
Find the sum: $z_1 + z_2 + z_3$ = 0

Find the product: $z_1 \cdot z_2 \cdot z_3$ = −8

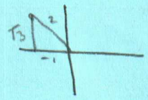
...ding Main Ideas

1) 

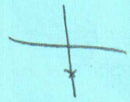
$(-2, 120^\circ)$ C point

2.) 

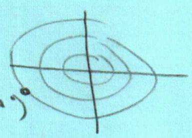
$(2, \pi)$ B point

3.) 

$2 \text{ cis } 120^\circ$ D complex #

4.) 

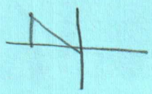
$2 \text{ cis } 270^\circ$ A complex #

5.) 

$(3, 200^\circ)$

$(3, -160^\circ)$

$(-3, 20^\circ)$

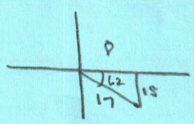
6.) 

$(4\sqrt{2}, 135^\circ)$

7.) $x = r\cos\theta$ $y = r\sin\theta$

$\left(-2\cos\frac{7\pi}{6}, -2\sin\frac{7\pi}{6}\right)$

$(1.73, 1)$

8.) 

$\tan^{-1}\left(\frac{15}{8}\right) = 61.9$

IV $\theta = 298^\circ$

$17 \text{ cis } 298^\circ$

9.)

10.) $\left(6 \text{ cis } \frac{7\pi}{12}\right)\left(\frac{1}{3} \text{ cis } \frac{\pi}{6}\right)$

$\left(6 \cdot \frac{1}{3}\right) \text{ cis } \left(\frac{7\pi}{12} + \frac{\pi}{6}\right)$

$2 \text{ cis } \frac{9\pi}{12} \quad = \quad 2 \text{ cis } \frac{3\pi}{4}$

$= 2(\cos 135 + i \sin 135)$

$= 2\left(\frac{-\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)$

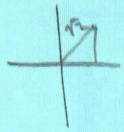
$= -\sqrt{2} + i\sqrt{2}$

11.) $a = r\cos\theta \qquad \frac{a}{r} = \cos\theta$

$b = r\sin\theta \qquad \frac{b}{r} = \sin\theta$ yes w/ substitution

$r = \sqrt{a^2+b^2}$

12.) $z = 1+i \qquad \sqrt{2} \text{ cis } 45 = z$



b.) $z^{-1} \qquad \frac{\sqrt{2}}{2} \text{ cis } 315^\circ \qquad \sqrt{2}^{-1} \text{ cis } 45 \cdot -1$

$= \frac{\sqrt{2}}{2} \text{ cis } -45$

c.) $z^2 = 2 \text{ cis } 90^\circ$

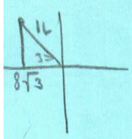
$(1+i)(1+i) = 1 + 2i + i^2 = 2i$

$2 \text{ cis } 90 = 2(\cos 90 + i \sin 90^\circ) =$

$= 2i$

13.) $-8 + 8i\sqrt{3}$

$16 \text{ cis } 150^\circ = z$



$(16 \text{ cis } 150)^{1/4} = z^{1/4}$

$16^{1/4} \text{ cis } \left(\frac{150}{4} + \frac{360k}{4}\right)$ 90° apart

$2 \text{ cis } 37.5$

127.5

$(8\cos 45, 8\sin -45) \rightarrow (4\sqrt{2}, -4\sqrt{2})$

table & symmetry

3. $-2i \rightarrow 2\text{cis}270°$ $= 2\sqrt{2}\text{ cis}(270+225)$

$-1-i \rightarrow \sqrt{2}\text{ cis }225°$ $= 2\sqrt{2}\text{ cis }495$

$= 2\sqrt{2}\text{ cis }135$

$\rightarrow -2+2i$

4. $(-1-i)^4 \rightarrow (\sqrt{2}\text{ cis}225)^4$

$= 4\text{cis}(225\cdot 4)$

$= 4\text{cis}900$

$= 4\text{cis}180$

5. $4i \rightarrow 4\text{cis }90$ $(4\text{cis}90)^{1/2}$

$\sqrt{2}+i\sqrt{2} \leftarrow 2\text{cis }\frac{90}{2} \Rightarrow 2\text{cis}45$

$-\sqrt{2}-i\sqrt{2} \leftarrow 2\text{cis }\frac{90}{2}+180 \Rightarrow 2\text{cis}225$

$(\sqrt{2}+i\sqrt{2})^2 = \cancel{2}+2i+2i+i^2\cancel{2}$

$= 4i$

$(\sqrt{2}-i\sqrt{2})^2 = \cancel{2}+2i+2i+\cancel{2i^2}$

$= 4i$

PT

1.) $(2, 90°)$ 2.) $(6\sqrt{2}, -45°)$ 3.) $(5, 143°)$

4.) $(4\cos 135, 4\sin 135)$ 5.) $(8\cos\frac{7\pi}{6}, 8\sin\frac{7\pi}{6})$ 6.) $(10\cos 310, 10\sin 310)$

$(-2\sqrt{2}, 2\sqrt{2})$ $(-4\sqrt{3}, -4)$ $(6.42, -7.66)$

7.) $b>a$ limacon w/ loop 8.) limacon w/o loop

Symmetric over $\theta=\pi/2$ min $y=-1$

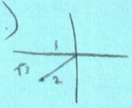
10.) spiral

$\boxed{11\text{-}16}$ $z_1 = 1 + i$ $\quad z_2 = -1 - i\sqrt{3}$ $\quad z_3 = \frac{3}{5} - \frac{4}{5}i$

11.) $|z_1| = \sqrt{1^2 + 1^2} = \sqrt{2}$

12.) $\sqrt{2}\ \text{cis}\ 45°$

13.)



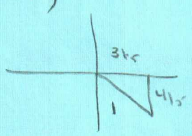
$2\ \text{cis}\ 240$

14.) $(\sqrt{2}\ \text{cis}\ 45)(2\ \text{cis}\ 240)$

$= 2\sqrt{2}\ \text{cis}\ (240 + 45)$

$= 2\sqrt{2}\ \text{cis}\ (285)$

15.)



$1\ \text{cis}\ 307°$ ← rounded

$= (1^{10}\ \text{cis}\ 307 \cdot 10)$

$= 1\ \text{cis}\ 3070$

$= -.984 + -.1736i$

16.) $(1\ \text{cis}\ 307)^{1/3} = 1^{1/3}\ \text{cis}\left(\frac{307}{3} + \frac{360k}{3}\right)$

$= 1\ \text{cis}\ 102 \qquad = -.21 + .978i$

$= 1\ \text{cis}\ 222 \qquad = -.743 + -.67i$

$1\ \text{cis}\ 342 \qquad = .95 + -.31i$