Now you try:

1. Look at the function for p(t) and m(t). How are the functions similar and different?

Similar b/c continuous change. Different b/c rate pos pop growth, negative for decay

2. How will the graphs be similar and different?

Scientists express the rate of decay in terms of half-life, the time required for half the mass to decay.

3. Write a formula for r in terms of t. Let $m_0 = 1$ and t is the time it will take for half of the initial mass to remain. Solve for r.

$\frac{1}{2} = e^{-rt}$ $\ln \frac{1}{2} = -rt$ $\boxed{r = \frac{-\ln(0.5)}{t}}$

4. Polonium-210 has a half-life of 140 days. Suppose a sample of this substance has a mass of 100 mg. find the mass remaining after one year.

$r = \frac{-\ln(0.5)}{140} = 0.4951\ \%$ $m(365) = 100(e^{-0.004951 \cdot 365})$
$= \boxed{16.41 \text{ grams}}$

Newton's Law of Cooling

An interesting use of logarithms and exponential functions involves determining how long it takes an object to cool. Newton's Law of Cooling is used to help determine how long someone discovered deceased has been deceased.

Newton's Law of Cooling- If D_0 is the initial temperature difference between an object and its surroundings, and if its surroundings have a temperature T_s, then the temperature of the object at time t is modeled by the function $T(t) = T_s + D_0e^{-kt}$ where k is a positive constant that depends on the type of object.

$T(25) = 85 + (210-85)e^{-0.041 \cdot 25}$
$= \boxed{129.85°F}$

Now you try:

5. A cup of coffee has a temperature of 210ºF and is placed in a room that has a temperature of 85ºF. After 20 minutes the temperature of the coffee is 140ºF. Find the temperature of the coffee after 25 minutes.

$T(20) = 140$

$140 = 85 + (210-85)e^{-k(20)}$
$55 = 125e^{-20k}$

$0.44 = e^{-20k}$ $K = 0.041$
$\ln 0.44 = -20k$

Adapted from "Precalculus Mathematics for Calculus, 5th edition," Stewart, 2009 pp. 369-382

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Logarithmic Scales

When a physical quantity varies over a very large range, it is more convenient to take the quantities logarithm in order to have a more manageable set of numbers. Below are a few examples of this process.

pH Scale

Chemists measured the acidity of a solution by giving its hydrogen ion concentration until Sorensen proposed a more convenient measure.

$$pH = -\log [H^+]$$

Where $[H^+]$ is the concentration of hydrogen ions measured in moles per liter (M). A pH of 7 is neutral, <7 is acidic and >7 basic.

Now you try:

6. The hydrogen ion concentration in a sample of human blood was measured to by $[H^+] = 5.41\times10^{-8}$M, find the pH and classify the blood as acidic or basic.

$pH = -\log(5.41 \times 10^{-8})$ Blood is Basic

$pH = 7.27$

Richter Scale

In 1935, Charles Richter defined the magnitude of an earthquake to be:

$$M = \log\frac{I}{S}$$

Where I is the intensity of the earthquake (measured by the amplitude of a seismograph reading taken 100 km form the epicenter of the earthquake) and S is the intensity of a "standard" earthquake (whose amplitude is 1 micron= 10^{-4}cm). The magnitude of a standard earthquake would be when the intensity=S so

$M = \log\frac{S}{S} = \log 1$=0

Adapted from "Precalculus Mathematics for Calculus, 5th edition," Stewart, 2009 pp. 369-382

N:

$S = 10^{-4}$ cm

Now you try:

I.

7. An earthquake in the United States has an estimated magnitude of 3.7 on the Richter scale. In the same year an earthquake in Ecuador was four times as intense. What was the magnitude of the Ecuador earthquake on the Richter scale?

$M_{us} = 3.7$

$3.7 = \log \frac{I}{10^{-4}}$

$= \log I - \log 10^{-4}$

$= \log I + 4$

$-0.3 = \log I$

$10^{-0.3} = I$

$M_{Ecuador} = \log \frac{4I}{S}$

$= \log \frac{4(10^{-0.3})}{10^{-4}}$

$= \log 4 + \log 10^{3.7}$

$\boxed{M_E = 4.3}$

The Decibel Scale

The ear is sensitive to a wide range of sound intensities. Scientist have determined that the intensity of sound that is just barely audible to by $I_0 = 10^{-12} W/m^2$ (watts per square meter) at a frequency of 1000 hertz. The psychological sensation of loudness varies with the logarithm of the intensity, so the intensity level B measured in decibels (dB) is:

$$B = 10 \log \frac{I}{I_0}$$

Now you try:

8. Find the decibel intensity level of a jet engine during takeoff if the intensity was measured at 10,000 W/m^2.

$B = 10 \log \frac{10{,}000}{10^{-12}}$

$= 10 \log \frac{10^4}{10^{-12}}$

$= 10 \log 10^{16}$

$= \boxed{160 \text{ dB}}$

Adapted from "Precalculus Mathematics for Calculus, 5th edition," Stewart, 2009 pp. 369-382

Practice Problems:

1. A fox population in a certain region has a relative growth rate of 8% per year. It is estimated that the population in 2000 was 18,000.

   a) Find a function that models the population t years after 2000.

   $P(t) = 18000(1 + 0.08)^t$

   b) Use the function from part (a) to estimate the fox population in year 2008.

   $= 18000(1.08)^8 = \boxed{33{,}316 \text{ fox}}$

   c) The region can only support 50,000 fox. In what year will the population surpass 50,000?

   $50{,}000 = 18000(1.08)^t$

   $\frac{50}{18} = 1.08^t$

   $t = \frac{\log(\frac{50}{18})}{\log(1.08)} = 13.3$ years

   $\boxed{\text{In } 2013}$

2. A culture contains 1500 bacteria initially and doubles every 30 minutes.

   a) Find a function that models the number of bacteria after t minutes.

   $B(t) = 1500(2)^{\frac{t}{30}}$

   b) Find the number of bacteria after 2 hours. $= 120$ min

   $B(120) = 1500(2)^{\frac{120}{30}} = \boxed{24{,}000 \text{ bacteria}}$

   c) After how many minutes will the culture contain 4000 bacteria?

   $4000 = 1500(2)^{\frac{t}{30}}$ $\quad \frac{40}{15} = 2^{t/30}$

   $t = 30\left(\frac{\log(\frac{40}{15})}{\log(2)}\right) = \boxed{42.5 \text{ min}}$

3. The half-life of radium-226 is 1600 years. Suppose we have a 22-mg sample.

   a) Find a function that models the mass remaining after t years.

   $m(t) = 22\left(\frac{1}{2}\right)^{\frac{t}{1600}}$

   b) After how long will only 18 mg of sample remain?

   $18 = 22\left(\frac{1}{2}\right)^{\frac{t}{1600}}$

   $\frac{9}{11} = 0.5^{t/1600}$

   $t = 1600\left(\frac{\log(\frac{9}{11})}{\log(0.5)}\right) = \boxed{463.2 \text{ years}}$

–5% remains

4. Radium-221 has a half-life of 30 seconds. How long will it take for 95% of the sample to decay?

$r(t) = r_0\left(\frac{1}{2}\right)^{t/30} \rightarrow 0.05 = \left(\frac{1}{2}\right)^{t/30}$

Adapted from "Precalculus Mathematics for Calculus, 5th edition," Stewart, 2009 pp. 369-382

$t = 30\left(\frac{\log(0.05)}{\log(0.5)}\right) = \boxed{130 \text{ seconds}}$

2 min + 10 seconds

$T(t) = T_s + D_0 e^{-kt}$ $T_s = 60°F$
$D_0 = 98.6°F - 60°F = 38.6°F$
$k = 0.1947$

5. Newton's Law of Cooling is used in homicide investigations to determine the time of death. The normal body temperature is 98.6ºF. Immediately following death the body begins to cool. It has been determined experimentally that the constant in Newton's Law of Cooling is approximately k=0.1947, assuming time is measured in hours. Suppose that the temperature of the surroundings is 60ºF.

a) Find a function that models the temperature t hours after death.

Let t = hours after death
T(t) = Temp after t hours

$T(t) = 60 + 38.6\, e^{-0.1947t}$

b) If the temperature of the body is now 70ºF, how long ago was the time of death?

$70 = 60 + 38.6\, e^{-0.1947t}$

$10 = 38.6\, e^{-0.1947t}$

$\frac{10}{38.6} = e^{-0.1947t}$

$t = \frac{\ln\left(\frac{10}{38.6}\right)}{-0.1947} = 6.94$ hours

TOD about 7 hours ago.

6. An unknown substance has a hydrogen ion concentration of $[H^+] = 3.1\times10^{-8}$M. Find the pH and classify the substance as acidic or basic.

$pH = -\log(3.1\times10^{-8})$
$= \boxed{7.5}$

$7.5 > 7 \rightarrow \boxed{\text{basic}}$

7. The 1906 earthquake in San Francisco had a magnitude of 8.3 on the Richter scale. At the same time in Japan an earthquake with a magnitude 4.9 caused minor damage. How many times more intense was the San Francisco earthquake than the Japanese earthquake?

$M_{SF} = 8.3 = \log\frac{I}{10^{-4}}$

$8.3 = \log I - \log 10^{-4}$

$8.3 = \log I + 4$

$4.3 = \log I \quad 10^{4.3} = I$

$M_J = 4.9 = \log\frac{I}{10^{-4}}$

$4.9 = \log I + 4$

$0.9 = \log I$

$I_J = 10^{0.9}$

About 2512 times more intense

8. The noise form a power mower was measured at 106 dB. The noise level of a rock concert was measured at 120 dB. Find the ratio of the intensity of the rock music to that of the power mower.

$B_M = 106\,dB = 10\log\frac{I}{10^{-12}}$

$106 = 10(\log I + 12)$

$106 = \log I^{10} + 120$

$-14 = \log I^{10}$

$10^{-14} = I^{10}$

$I = 10^{-14/10}$

$I = 0.04\ W/m^2$

$B_C = 120\,dB = 10\log\frac{I}{10^{-12}}$

$120 = 10(\log I - \log 10^{-12})$

$120 = \log I^{10} + 120$

$0 = \log I^{10}$

$10^0 = I^{10}$

$1 = I$

$1\ W/m^2$

$\boxed{1 : 0.04 = 25}$

Adapted from "Precalculus Mathematics ~~for Calculus~~, 5th edition," Stewart, 2009 pp. 369-382