Forming Functions from Word Problems
Section 4-7

1. Express the area A of an equilateral triangle as a function of the perimeter P.

$A = \frac{\sqrt{3}}{4}s^2$

$P = 3s$

$s = \frac{P}{3}$

Substitute $s = \frac{P}{3}$ → $A(P) = \frac{\sqrt{3}}{4}\left(\frac{P}{3}\right)^2 = \frac{\sqrt{3}\,P^2}{36}$

2. A store owner bought n dozen toy boats at a cost of \$3 per dozen and sold them at \$0.75 a piece. Express the profit (in dollars) as a function of n. Let n = dozen toy boats

$C = \$3n$

Want $P(n) = ?$

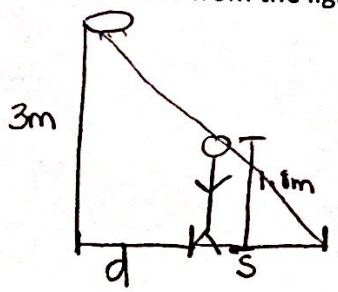
$R = \$0.75 / 1 \text{ boat} \rightarrow \$0.75 \times 12 = \$9 \text{ per dozen boats} = 9n$

Profit = Revenue − Cost

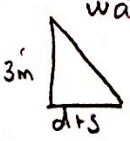
$P(n) = 9n - 3n$

$P(n) = 6n$

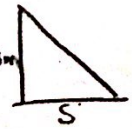
3. A light 3m above the ground causes a boy 1.8m tall to cast a shadow s meters long measured along the ground. Express s as a function of d, the boy's distance in meters from the light.



Want $s(d) = ?$



- similar to



$\Rightarrow \frac{3m}{1.8m} = \frac{d+s}{s}$

$3s = 1.8d + 1.8s$

$1.2s = 1.8d$

$\boxed{s(d) = 1.5d}$

4. The height of a cylinder is twice the diameter. Express the total surface area as a function of the height h.

① $A(h) = ?$

$A = 2\pi r^2 + 2\pi rh$

↓ 2 bases are circles; ↓ surface area of body.

Know $h = 2d$, $d = 2r$

③ Put $2\pi r^2$ in terms of h.

$h = 2d = 2(2r)$

$h = 4r$

$\frac{h}{4} = r$

$2\pi r^2 = 2\pi\left(\frac{h}{4}\right)^2$

Put $2\pi rh$ in terms of h

$2\pi rh = 2\pi\left(\frac{h}{4}\right)h$

④ $A(h) = 2\pi\left(\frac{h}{4}\right)^2 + 2\pi\left(\frac{h}{4}\right)h$

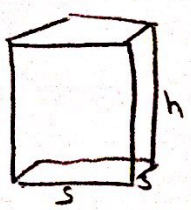
$= 2\pi\frac{h^2}{16} + 2\pi\frac{h^2}{4}$

$= \frac{\pi h^2}{8} + \frac{4\pi h^2}{8}$

$\boxed{A(h) = \frac{5}{8}\pi h^2}$

5. A box with a square base and no top has a volume of 6 meters cubed. Express the total surface area A of the box as a function of the width of the base.

① $A(s) = ?$



$V = s^2h = 6\,m^3$

$A = s^2 + 4sh$

③ solve for h using volume

$6 = s^2h$

$h = \frac{6}{s^2}$

④ substitute h in Area function

$A(s) = s^2 + 4s\left(\frac{6}{s^2}\right) = s^2 + \frac{24}{s} = \frac{s^3 + 24}{s}$

$\boxed{A(s) = \frac{s^3 + 24}{s}}$