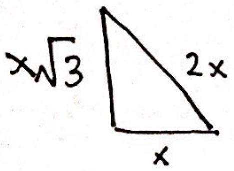
Date:

More Forming Functions from Word Problems
Worksheet #2
Section 4-7

1. Express the area A of a 30-60-90 triangle as a function of the length z of the hypotenuse. $\rightarrow z = 2x$, $x = \frac{z}{2}$

$A(z) = ?$



$A = \frac{1}{2} b \cdot h = \frac{1}{2} \cdot x \cdot x\sqrt{3}$ substitute $x = \frac{z}{2}$

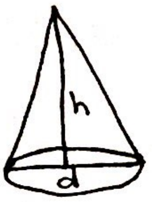
$b = x$ $\quad = \frac{1}{2}\left(\frac{z}{2}\right)\left(\frac{z}{2}\right)\sqrt{3}$

$h = x\sqrt{3}$ $\quad = \frac{z^2\sqrt{3}}{8}$

$A(z) = \frac{z^2\sqrt{3}}{8}$

2. A pile of sand is in the shape of a cone with a diameter that is twice the height. Express the volume V of sand as a function of the height h. ($V = \pi r^2 \frac{h}{3}$)

$V(h) = ?$



$d = 2h$ $\quad V = \frac{1}{3}\pi r^2 h$

$d = 2r$ $\quad = \frac{1}{3}\pi (h)^2(h)$ substitute $r = h$

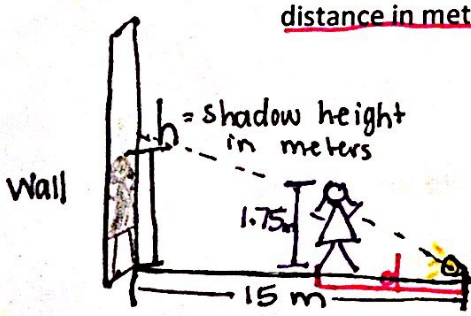
Therefore $\rightarrow 2h = 2r$ $\quad = \frac{h^3\pi}{3}$

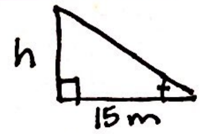
$h = r$

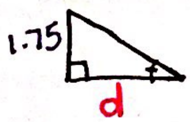
$V(h) = \frac{h^3\pi}{3}$

3. When a girl 1.75m tall stands between a wall and a light on the ground 15m away, she casts a shadow h meters high on the wall. Express h as a function of d, the girl's distance in meters from the light.

$h(d) = ?$







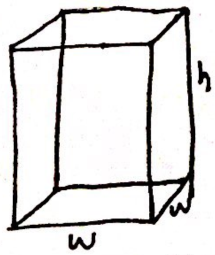
$\frac{1.75m}{h} = \frac{d}{15m}$

$26.25 = hd$

$h = \frac{26.25}{d} = 26\frac{1}{4} \div d = \frac{105}{4d}$

$h(d) = \frac{105}{4d}$

4. A box with a square base and no top has volume $8m^3$. The material for the base costs \$8 per square meter, and the material for the sides costs \$6 per square meter.
   a) Express the cost C of the materials used to make the box as a function of the width of the base. $C(w) = ?$



$V = w^2 h = 8m^3$ $\quad$ Surface Area $\rightarrow m^2$

$8 = w^2 h$ $\quad$ Area of base $\rightarrow w^2$ $\quad$ Cost material:

$h = \frac{8}{w^2}$ $\quad$ Area of sides $\rightarrow 4wh$ $\quad$ Base $= 8w^2$, Sides $= 4wh$

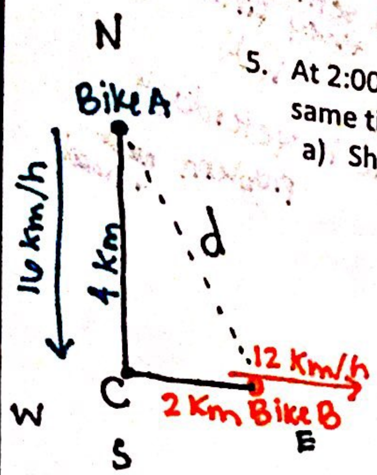
Total Cost $= 8w^2 + 4wh$

$\rightarrow$ substitute $\rightarrow = 8w^2 + 4w\left(\frac{8}{w^2}\right) = 8w^2 + \frac{192}{w} = \frac{8w^3 + 192}{w}$

   b) Use your graphing calculator to find the minimum cost.

Minimum cost approximately \$126.

$C(w) = \frac{8w^3 + 192}{w}$



5. At 2:00PM bike A is 4km north of point C and traveling south at 16 km/h. At the same time, bike B is 2km east of C and traveling east at 12km/h.

a) Show that t hours after 2:00PM the distance between the bikes is: $\sqrt{400t^2 - 80t + 20}$

$d_{\text{Bike A}} = 4 - 16t$

$d_{\text{Bike B}} = 2 + 12t$

$d^2 = d_A^2 + d_B^2$

$d = \sqrt{d_A^2 + d_B^2}$

$d = \sqrt{(4-16t)^2 + (2+12t)^2}$

$= \sqrt{16 - 128t + 256t^2 + 4 + 48t + 144t^2}$

$d(t) = \sqrt{400t^2 - 80t + 20}$

b) At what time is the distance between the bikes the least? Can trace on graphing calc or

min of $d(t)$ = min of $f(t) = 400t^2 - 80t + 20$

$V_t = \frac{-b}{2a} = \frac{80}{800} = \frac{1}{10}$

t = hours after 2:00 PM

$V_t = \frac{1}{10}$ of an hour = 6 min

Distance b/w bikes is the least at 2:06 P.M.

c) What is the distance between the bikes when they are closest?

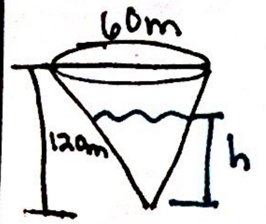
Least distance = closest

$d(\frac{1}{10}) = \sqrt{400(\frac{1}{10})^2 - 80(\frac{1}{10}) + 20} = \sqrt{4 - 8 + 20} = \sqrt{16} = 4$ km

4 km between bikes when they are closest.

6. A conical tank has a diameter of 60m and height of 120m. Water is flowing into the tank at a rate of $5m^3/s$.

$V = \frac{1}{3}\pi r^2 h$



a) Find the volume V of the water as a function of the water level h. $V(h) = ?$

d = 60m then r = 30m

$\frac{30}{120} = \frac{r}{h}$

$\frac{1}{4} = \frac{r}{h}$

$h = 4r \rightarrow r = \frac{h}{4}$

$V(h) = \frac{1}{3}\pi h \left(\frac{h}{4}\right)^2$

$= \frac{1}{3} \cdot \frac{h^3}{16} \pi$

$V(h) = \frac{1}{48} h^3 \pi$

b) Find h as a function of the time t during which water has been flowing into the tank. $h(t) = ?$

$\frac{5m^3}{s} \cdot t$ sec = Volume of water in take after t seconds

$V = \frac{1}{48} h^3 \pi$

$\sqrt[3]{\frac{48V}{\pi}} = h$

$h(t) = \sqrt[3]{\frac{48 \cdot 5t}{\pi}}$

$h(t) = \sqrt[3]{\frac{240t}{\pi}}$