Section 3.4

#23 Find a quartic equation w/ integral coefficients with roots $5-i\sqrt{3}$ & i

$\therefore$ other roots are $5+i\sqrt{3}$ & $-i$

$(x+i)(x-i) \rightarrow x^2-i^2 \rightarrow (x^2+1)$

$(x-(5+i\sqrt{3}))(x-(5-i\sqrt{3})) \rightarrow x^2-10x+28$

or use sum of roots $5+i\sqrt{3}+5-i\sqrt{3} = 10 = \frac{a_{n-1}}{a_n}$

product of roots $(5+i\sqrt{3})(5-i\sqrt{3}) = 28 = \frac{a_0}{a_n}$

$\therefore$ quadratic is $x^2-10x+28$

Quartic $(x^2+1)(x^2-10x+28)$

$= x^4-10x^3+28x^2+x^2-10x+28$

$= x^4-10x^3+29x^2-10x+28$

#25 Find integers c & d $x^3+cx+d=0$ has $1+\sqrt{3}$ as one root

$\therefore$ 2^{nd} root is $1-\sqrt{3}$, 3^{rd} root is r (unknown)

Sum of roots $1+\sqrt{3}+1-\sqrt{3}+r=0$ (since coefficient of x^2 is 0)

$2+r=0$

$r=-2$

product of roots $= \frac{-a_0}{a_n}$: $-2(1+\sqrt{3})(1-\sqrt{3}) = -2(1-3) = 4 = \frac{-a_0}{a_n}$

$\therefore a_0=-4$

& $\boxed{d=-4}$

Substitute $x^3+cx-4=0$

let $x=-2$ $(-2)^3-2c-4=0$

$-8-2c-4=0$

$-12=+2c$

$\boxed{-6=c}$

#28 Find d such that $x^3+4x^2-9x+d=0$ has 2 roots that are that are additive inverses of each other

Let 1^{st} root $=r$ 2^{nd} root $=-r$ (so additive inverses) 3^{rd} root $=p$

Sum of roots $\longrightarrow$ $r+-r+p = \frac{-4}{1}$ $\therefore p=-4$

Substitute to solve for d