Name: _____ **614 Polynomial Review Sheet**

1. Use synthetic division to find P(−2) if $x^5+2x^2-5x^3+9x=P(x)$

$P(x)=x^5-5x^3+2x^2+9x$

-2 | 1 0 −5 2 9 0
−2 4 2 −8 −2
1 −2 −1 4 1 −2

2. Find all the zeros of $x^3+9x^2+17x+6=P(x)$

PRR = $\frac{\pm1\pm2\pm3\pm6}{\pm1}$ $\Rightarrow \pm1\pm2\pm3\pm6$

-2 | 1 9 17 6
−2 −14 −6
1 7 3 0

$x^2+7x+3=0$

$x=\frac{-7\pm\sqrt{49-4(1)(3)}}{2}$

$=\frac{-7\pm\sqrt{37}}{2}$, -2

3. Factor the polynomial expression: x^3+5x^2+2x-8

PRR = $\pm1\pm2\pm4\pm8$
Sum = −5
Product = 8

1 | 1 5 2 −8
1 6 8
1 6 8 0

x^2+6x+8
$(x+4)(x+2)$
Zeros: 1, −2, −4

4. If a polynomial function has zeros of −2, 3/5, 3 and −1
   a.) What are the factors of the function? $(x+2)(x-\frac{3}{5})(x-3)(x+1)$
   b.) What is the degree of the function? 4th degree

or $(x+2)(5x-3)(x-3)(x+1)$

5. Determine the factors of the polynomial function $f(x)=4x^3-11x^2+x+1$

PRR = $\pm1\pm\frac{1}{2}\pm\frac{1}{4}$

$-\frac{1}{4}$ | 4 −11 1 1
−1 3 −1
4 −12 4 0

$4x^2-12x+4$ $4(x^2-3x+1)$

$x=\frac{3\pm\sqrt{9-4(1)(1)}}{2}$

$=\frac{3\pm\sqrt{5}}{2}$

2 roots: $\frac{3+\sqrt{5}}{2}$, $\frac{3-\sqrt{5}}{2}$, $-\frac{1}{4}$

6. If 2 is a zero of the polynomial $f(x)=3x^2+kx-8$, find the value of k.

2 | 3 k −8
6 12+2k
3 6+k 4+2k

$4+2k=0$
$2k=-4$
$k=-2$

7. Two roots of the equation $x^4-3x^3-14x^2+12x+40=0$ are x=2 and x=5. Find the remaining roots.

2 | 1 −3 −14 12 40
2 −2 −32 −40
5 | 1 −1 −16 −20 0
5 20 20
1 4 4 0

x^2+4x+4
$(x+2)(x+2)$

4 roots
2, −2 (dr), 5

8. Determine a polynomial function with real coefficients of lowest degree and leading coefficient of 1 that has 1+5i and 2 as zeros.

$P(x)=x^3-4x^2+9x-52$

Sum $1+5i+1-5i+2=4=-a_{n-1}$ $a_{n-1}=-4$

Product $2(1+5i)(1-5i)=2(1-25i^2)=52=\frac{-a_0}{a_n}$

$P(x)=x^3-4x^2+9x-52$
$P(2)=0$
$a_0=-52$

$0=8-16+18-52$
$26=60$
$a=30$

9. Determine the zeros of the polynomial function $p(x)=x^5+x^3-2x^2-12x-8$

−1 | 1 0 1 −2 −12 −8
−1 1 −2 4 8
1 −1 2 −4 −8 0

−1 | 1 −1 2 −4 −8
−1 2 −4 8
1 −2 4 −8 0

$x^3-2x^2+4x-8=0$
$x^2(x-2)+4(x-2)=0$
$(x^2+4)(x-2)=0$
$x=2\pm2i$
roots: $\pm2i, 2, -1$ dr

10. Find a cubic equation with integral coefficients having 3i and (−5/2) as roots.

Sum $3i+-3i+-5/2=-5/2=\frac{-a_{n-1}}{a_n}$ $a_{n-1}=+5$ $a_n=2$

Product $(3i)(-3i)(-5/2)=\frac{+45i^2}{2}=-\frac{45}{2}=\frac{-a_0}{a_n}$ $a_0=-45$

$P(x)=2x^3+5x^2+cx-45$
$P(-5/2)=0$
$\frac{5}{2}c=45$ $c=18$
$c=18$ by substitution

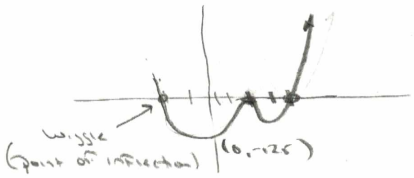
11. $P(x)=x^3+5x^2+x-3$ has no positive rational roots but it does have a positive irrational root. Using the location principle, between what to consecutive integers will this positive root be found? Justify your answer.

$P(0) = -3$

$P(1) = 1+5+1-3 = 4$

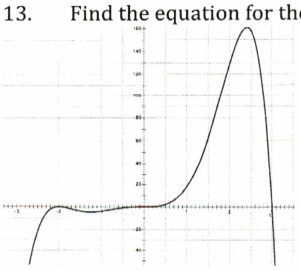
Sign changes on y-value from negative to positive ∴ a root

12. Sketch the graph of $y=(x-3)^2(x+2)(x-5)^3$



degree 6
a > 0
↑ ↑
y-intercept $4 \cdot 2 \cdot 125$

13. Find the equation for the graph.



degree 4
Dr -2
0
3

$P(x) = -x(x+2)^2(x-3)$

14. A life guard marks off a rectangular swimming area at a beach with 200 m of rope. What is the greatest area of water she can enclose? What are the dimensions of the area?

x → width of area

$A(x) = x(200-2x)$

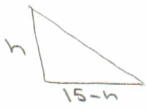
$= 200x - 2x^2$

length $200-2(50) = 100$ m

50 m × 100 m

$\text{Vertex} = \frac{-b}{2a} = \frac{-200}{-4} = 50$

15. What is the maximum area of a triangle having 15 cm as the sum of its base and height? Identify your variables; create an equation and solve.



$A = \frac{1}{2}b \cdot h$

$A(h) = \frac{1}{2}(15-h)h$

$= \frac{1}{2}(15h-h^2)$

$= \frac{-h^2}{2} + \frac{15}{2}h$

$V_x = \frac{-\frac{15}{2}}{2(-\frac{1}{2})} = \frac{-15}{2} \cdot -1 = 7.5$ cm

16. Determine a polynomial equation with degree 5 that has two imaginary roots and at least one irrational root. How many rational roots must this equation have? Why?

1 rational root
2 irrational roots (pair)
2 imag roots