Questions
8b)

Name: _____

More Exponents and Logarithms

1. Simplify each expression. The properties may help you.

a.) $\log_5 \frac{1}{5}$

$\log_5 5^{-1}$

-1

b.) $\ln e^x$

x

c.) $\ln e^{x+1}$

$x+1$

d.) $\log_3 \frac{1}{\sqrt{3}}$

$\log_3 3^{-\frac{1}{2}}$

$-\frac{1}{2}$

e.) $\log 10^{-8}$

-8

f.) $\log \sqrt{1000}$

$\log 10^{3/2}$

$3/2$

2. Solve each logarithmic equation.

a.) $\log_3(x+1)=4$

$3^4 = x+1$

$81 = x+1$

$x = 80$

b.) $\log_2(3x-4)=5$

$2^5 = 3x-4$

$32 = 3x-4$

$x = 12$

c.) $\log_2(4x-1)=\log_2 2$

$4x-1 = 2$

$4x = 3$

$x = \frac{3}{4}$

d.) $\log\frac{x}{x-1}=\log\frac{1}{2}$

$\frac{x}{x-1} = \frac{1}{2}$

$2x = x-1$

$x = -1$

3. Solve each exponential equation for x.

a.) $10^{x+1} = 9$

$\log$

$(x+1)\log 10 = \log 9$

$x+1 = \log 9$

$x = \log 9 - 1$

$x \approx -0.0458$

b.) $2500 = 1000e^{x-4}$

$2.5 = e^{x-4}$

$\ln 2.5 = x-4$

$x = \ln 2.5 + 4$

$x \approx 4.9163$

c.) $200 = 400e^{-0.02x}$

$\frac{1}{2} = e^{-0.02x}$

$\ln(0.5) = -0.02x$

$x = \frac{\ln(0.5)}{-0.02}$

$x \approx 34.6574$

d.) $32 = 8 \cdot (10^{6x-4})$

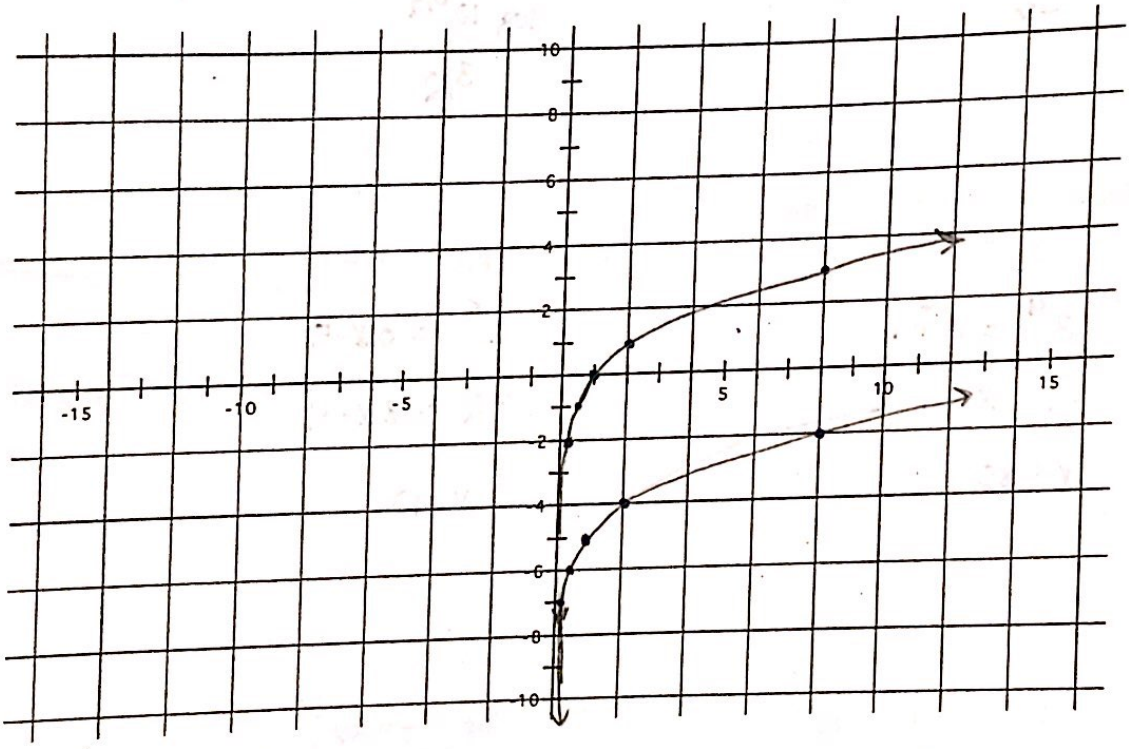
$4 = 10^{6x-4}$

$\log 4 = 6x - 4$

$\log 4 + 4 = 6x$

$x = \frac{\log 4 + 4}{6}$ $x \approx 0.767$

4. Sketch the graph of $f(x) = \log_2 x$. Explain HOW the graph of $g(x) = \log_2 x - 5$ compares with its parent. Find the domain for each function.



$f^{-1}(x) = 2^x$

$f(x) = \log_2 x$

D: $x > 0$

R: $y \in \mathbb{R}$

$g(x) = \log_2 x - 5$

downward vertical shift of 5 units.

D: $x > 0$

R: $y \in \mathbb{R}$

5. Solve for x. Remember to check for extraneous solutions. An extraneous solution is a solution to the rewritten equation but not the original equation. Some solutions to the rewritten equations will cause logarithms of o or negative numbers. Check all solutions in the original equation to make sure they are valid.

a.) $\log_2(x^2+3x-10)=3$

$2^3 = x^2+3x-10$

$0 = x^2+3x-18$

$0 = (x+6)(x-3)$

$x=-6$ or $x=3$

Check:

$x=-6$: $\log_2((-6)^2+3(-6)-10) \stackrel{?}{=} 3$

$\log_2 8 = 3$ ✓

$x=3$: $\log_2(3^2+3(3)-10) \stackrel{?}{=} 3$

$\log_2 8 = 3$ ✓

b.) $\log_5(x^2+5x-4)=\log_5(x+1)$

$x^2+5x-4 = x+1$

$x^2+4x-5=0$

$(x+5)(x-1)=0$

$x=-5$ or $x=1$

Check:

$x=-5$: $\log_5((-5)^2+5(-5)-4) \stackrel{?}{=} \log_5(-5+1)$

$\log_5(-4) = \log_5(-4)$

extraneous → can't take log of negative #

$x=1$: $\log_5(1+5-4) \stackrel{?}{=} \log_5(1+1)$

$\log_5 2 = \log_5 2$ ✓

6. Rewrite as a single logarithm

a.) $\log_2 3x - 4\log_2 y$

$\log_2 3x - \log_2 y^4$

$\log_2\left(\frac{3x}{y^4}\right)$

b.) $3\log 4x + 2\log 3 - 2\log y$

$\log(4x)^3 + \log 9 - \log y^2$

$\log\left(\frac{64x^3 \cdot 9}{y^2}\right)$

$\log\left(\frac{576x^3}{y^2}\right)$

7. Expand each logarithm.

a.) $\log\frac{4x}{y}$

$\log 4x - \log y$

$\log 4 + \log x - \log y$

b.) $\log_4\frac{10}{\sqrt{4x}} = \log_4\frac{10}{(4x)^{\frac{1}{2}}}$

$\log_4 10 - \log_4(4x)^{\frac{1}{2}}$

$\log_4 10 - \frac{1}{2}\log_4 4x$

8. Solve each equation. Use properties to simplify your work.

a.) $\log_2(x-5)+\log_2(x+2)=3$

$\log_2(x-5)(x+2)=3$

$2^3 = x^2-3x-10$

$0 = x^2-3x-18$

$0 = (x-6)(x+3)$

$x=6$ or $x=-3$

Check: $x=6$

$\log_2(1)+\log_2(8) \stackrel{?}{=} 3$

$0+3=3$ ✓

$x=-3$

$\log_2(-8)+\log_2(-1) \stackrel{?}{=} 3$

Extraneous → can't take log of negative #

b.) $2\log_7(x+1)=2$

$\log_7(x+1)^2 = 2$

$\sqrt{49} = \sqrt{(x+1)^2}$

$\pm 7 = x+1$

$x=-8$ or $x=6$

Check: $x=-8$

$2\log_7(-8+1) \stackrel{?}{=} 2$

$2\log_7(-7) \neq 2$

extraneous

$x=6$

$2\log_7(6+1) \stackrel{?}{=} 2$

$2\log_7(7) = 2$ ✓

9. Solve each equation.

a.) $4^x = 5^{x-1}$

$\log 4^x = \log 5^{x-1}$

$x \log 4 = (x-1)(\log 5)$

$x \log 4 = x \log 5 - \log 5$

$x \log 4 - x \log 5 = -\log 5$

$x(\log 4 - \log 5) = -\log 5$

$x = 7.2126$

$x = 7.2126$

b.) $10^{2-x} = 5^{x+3}$

$(2-x)\log 10 = (x+3)\log 5$

$2 - x = x \log 5 + 3 \log 5$

$2 - 3\log 5 = x \log 5 + x$

$2 - 3\log 5 = x(\log 5 + 1)$

$x = \dfrac{2 - 3\log 5}{\log 5 + 1}$

$X = -0.057$

FOR ALL WORD PROBLEMS YOU MUST INCLUDE A FORMULA THAT DESCRIBES THE SITUATION AS WELL AS A SOLUTION. Intermediate steps are encouraged!

10. How long will it take for $1000 to grow to $1500 if it earns 8% annual interest compounded monthly?

$A(t) = 1000\left(1 + \dfrac{0.08}{12}\right)^{12t}$

$1500 = 1000\left(1 + \dfrac{0.08}{12}\right)^{12t}$

$\dfrac{3}{2} = (1.0067)^{12t}$

$\log\left(\dfrac{3}{2}\right) = 12t \log(1.0067)$

$12t = 61.0223$

$t = 5.0852$

It will take About 5 years

11. A school district estimates that its student population will grow about 5% per year for the next 15 years. How long will it take the student population to grow from the current 8000 students to 12000? Assume continuous growth.

$P(t) = P_0 e^{rt}$

$12000 = 8000 e^{(0.05)t}$

$1.5 = e^{0.05t}$

$\ln 1.5 = \ln e^{0.05t}$

$0.05t = \ln 1.5$

$t = 8.1093$

About 8 years

12. The population of a certain city in 2004 is about 650,000. If it is losing 2% of its population each year, when will the population decline to 500,000?

In 2017

$P(t) = 650{,}000(0.98)t$

$500{,}000 = 650{,}000(0.98)t$

$\dfrac{500}{650} = 0.98t$

$\log\left(\dfrac{500}{650}\right) = t \log 0.98 \longrightarrow t = 12.9866$

13. Suppose a bacteria culture contains 2500 bacteria at 1:00 and at 1:30 there are 6000. What is the hourly growth rate? Assume continuous growth.

$8(t) = 2500 e^{rt}$

$6000 = 2500 e^{r(0.5)}$

$2.4 = e^{\frac{r}{2}}$

$\ln 2.4 = \dfrac{r}{2} \ln e \longrightarrow r = 1.7509$

Hourly growth rate = 175.09%

$r = 1.7509$

Hourly growth rate 175.09%

14. A corporation that owns a chain of retail stores operated 500 stores in 2000 and 700 stores in 2003. Assuming that the number of stores is growing exponentially, what is the annual growth rate?

$700 = 500(1+r)^3$

$\left(\frac{7}{5}\right)^{\frac{1}{3}} = 1+r$

$r = 0.1187$

11.87 % annual growth rate

15. The half-life of carbon-14 is approximately 5700 years therefore an equation that would describe the amount of carbon-14 left after t years is $A(t) = A_0(\frac{1}{2})^{\frac{t}{5700}}$

r = 0.1187

11.87 % annual growth rate

a.) How long will it take for 80% of the carbon-14 to decay in an animal after it has died?

$0.2 = (0.5)^{t/5700}$

$\log(0.2) = \frac{t}{5700}\log(0.5)$

$t = 13,234.99$

13,235 years

b.) Suppose an animal dies today. How much of its carbon-14 will remain after 250 years?

$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{250}{5700}}$

$\frac{A(t)}{A_0} = 0.97$

97 % of carbon-14 will remain