(25) $h(x) = \frac{x}{x-1}$

$$h^{-1}(x) = x = \frac{y}{y-1}$$

$$x(y-1) = y$$

$$xy - x = y$$

$$xy = y + x$$

$xy - y = x$ ← What do you notice?

$$y(x-1) = x$$

$$y = \frac{x}{x-1}$$

$$h^{-1}(x) = \frac{x}{x-1}$$

★ Note inverse is the same as the original function.

Think back to ~~our unit~~ section on reflections and symmetry. If inverse = original function, then the graph has a line of symmetry about the line $y = x$.

(16) see graph

Inverse of $f(x) = \sqrt{5-x}$ is a function.

→ Graph of $f(x)$ passes horizontal line test

$f^{-1}(x) = x = \sqrt{5-y}$

$x^2 = 5-y$

$y + x^2 = 5$

$y = 5 - x^2$

Domain of $f^{-1}(x)$ is the RANGE of $f(x)$

$f(x) = \sqrt{5-x}$ Range $= y \geq 0$ (see graph)

Domain $= 5$

↓

Becomes DOMAIN of $f^{-1}(x) \rightarrow x \geq 0$

Algebraic proof that $f^{-1}(x)$ is inverse of $f(x)$

$f(f^{-1}(x)) = f^{-1}(f(x)) = x$

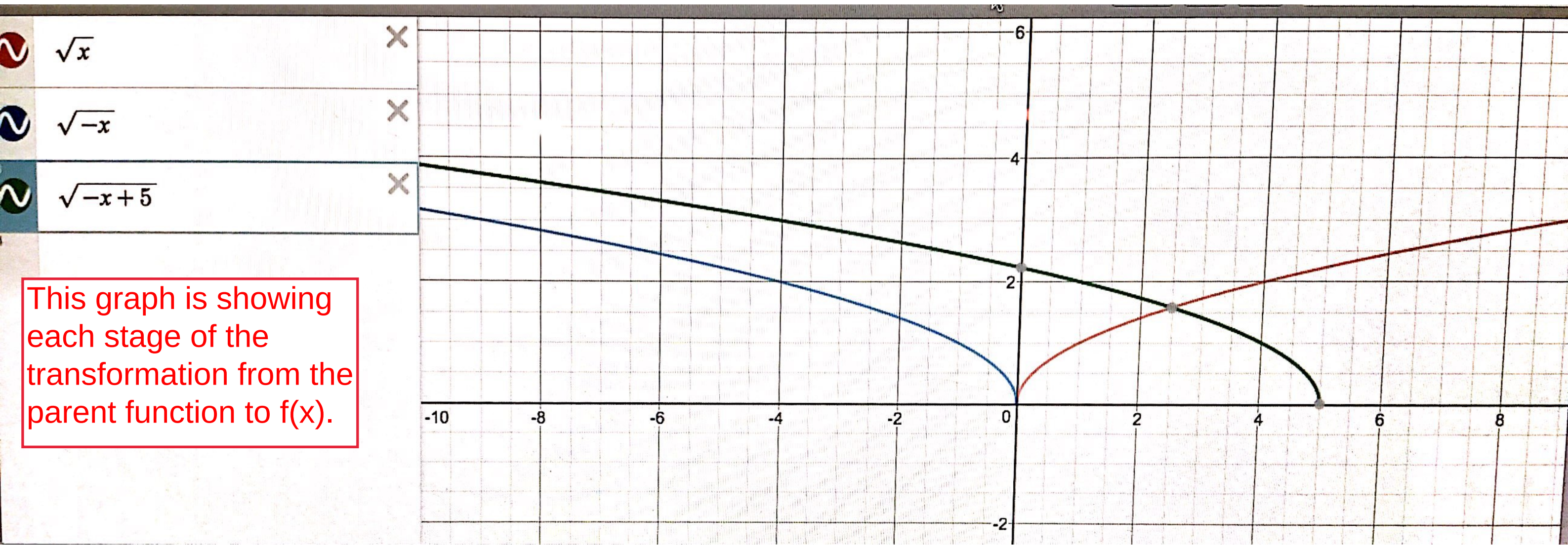
$f(5-x^2) = f^{-1}(\sqrt{5-x}) = x$

$\sqrt{5-(5-x^2)} = 5-(\sqrt{5-x})^2 = x$

$\sqrt{5-5+x^2} = 5-(5-x) = x$

$\sqrt{x^2} = 5-5+x = x$

$x = x = x$ ✓



This graph is showing each stage of the transformation from the parent function to f(x).